Path Length Variation in a Synchronous Satellite Communications Link

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The path length and path length rate variations in a communications link connecting earth stations on the East and West coasts of the continental United States by means of an equatorial synchronous satellite are investigated numerically and shown to be less than 50,000 feet and 5 feet per second for periods of at least a day (and likely for a week or more) with no station-keeping during the periods of interest.

I. PROBLEM

The use of a synchronous satellite in a communications link with time division switching for digital channels may introduce problems because of the path length variations resulting from the perturbation of the satellite orbit. For instance, a path length variation of 0.02 percent (10 miles for a typical satellite-tracker configuration) corresponds to a timing change of 0.05 ms. The rate of change of path length (length rate) is also of interest. The object of this paper is not to apply the results to current problems, but to make the results available to prospective users.

Analyses of the synchronous satellite problem[1,2] have emphasized the long period motion. However, the relatively small short period perturbations have important effects upon path length and length rate. Also we expect the long period motion to be handled by discrete, rather than continuous, station keeping (perhaps by a sequence of impulsive maneuvers every few days) so that the major orbital perturbations will be those of short period.

We assume an equatorial orbit, with the longitude fixed by the communication requirements at 100°W. We must specify the initial altitude and velocity so that the satellite is synchronous. If we assume that the earth's inverse-square gravity results in a circular equatorial orbit with mean motion equal to the earth's rotation rate, then it can

be shown that perturbative forces such as the J_2, J_3, J_4 zonal harmonics and the J_{22} tesseral harmonic introduce radial and tangential forcing terms (e and n) into the equations of motion.[2]

II. ANALYSIS

If the initial satellite longitude is equal to the longitude of the major axis of the earth's elliptical equator, then the initial radius, r_s, can be chosen so e and n are both equal to zero (if higher order terms are neglected). If the initial longitude must be different, then $n = 0$ cannot be obtained and there will be drift away from the desired longitude. However, we can still obtain $e = 0$ by proper choice of r_s (with little effect upon n) and this is the procedure we follow.

We do not include the luni-solar gravity perturbations (which we include in our computer runs) in the choice of initial conditions, since these perturbations are time-varying in an earth-fixed frame and the details of their inclusion depend upon the interval of interest.[1] Their effect is small, and the complexity they introduce into an operational procedure probably outweighs any reduction in the perturbations.

Table I—Path Length and Length Rate Variations

Date	$L_{MAX} - L_{MIN}$ (feet)	$\dot{L}_{MAX} - \dot{L}_{MIN}$ (feet per second)
1–01:00	16,200	1.45
1–21:16*	30,700	2.69
4–01:00	27,000	2.58
4–20:21*	32,400	2.88
7–01:00	35,600	3.40
7–18:05*	31,800	3.12
8–16:12*	40,700	4.24
9–14:19*	38,900	4.03
10–01:00†	36,100	3.71
10–08:00	12,600	1.27
10–14:04*	35,900	3.56
10–15:00	41,300	4.22
10–22:00	12,900	1.14
10–29:00‡	38,400	3.85
11–12:14*	45,200	4.15
12–12:03*	29,700	2.75
12–22:00	18,900	1.65

* New moon
† Full moon on 9–29:17
‡ Full moon on 10–29:10

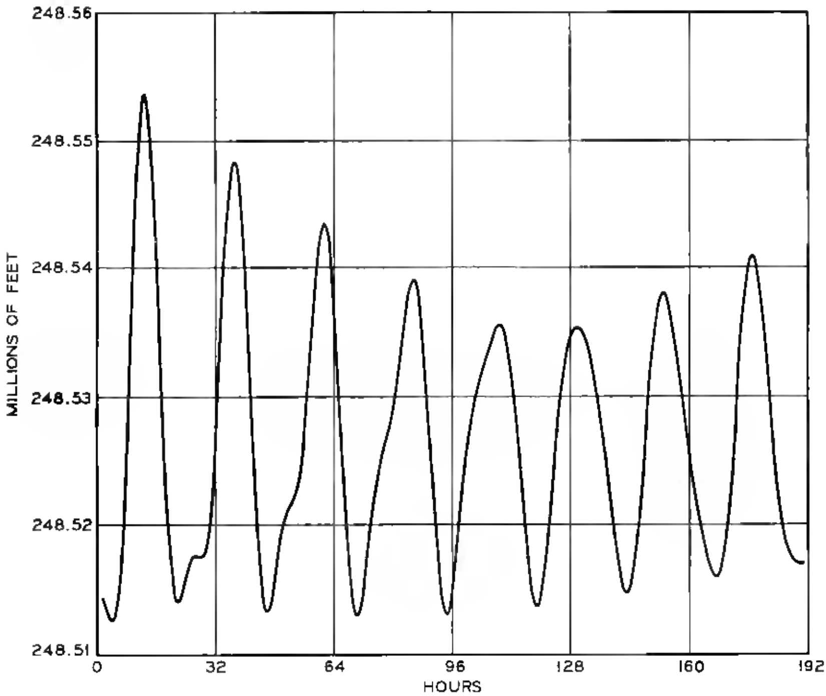

Fig. 1 — Path length variation for eight days starting 10–15:00.

III. RESULTS

The results were obtained using modifications of existing computer programs to integrate the equations of motion of the satellite numerically.[3]

The ground stations are located at 38°N latitude and at 75°W and 125°W longitude. The initial satellite longitude is midway between the tracker longitudes. We list some results in Table I. In each case the satellite has been injected into orbit, or the orbit has been corrected, so that the radial forcing function is zero on the date shown. (The dates are given as "month-day:hour," so that 1–21:16 means January 21 at 4 p.m., Greenwich time. All runs were made for 1966 since simple luni-solar position routines were available for that year; however, they should be quite representative of the variations to be expected.)

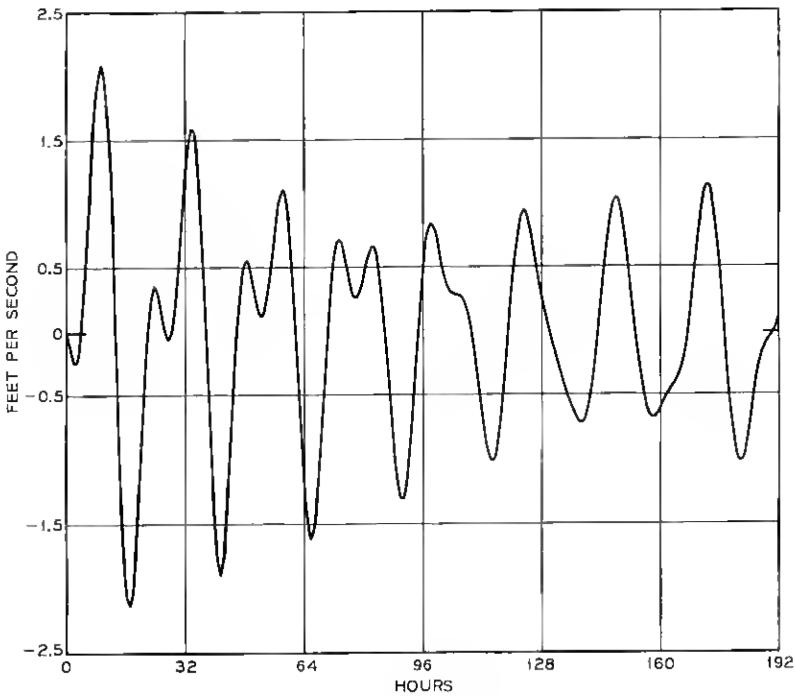

Fig. 2—Path length rate variation for eight days starting 10–15:00.

The maximum path length and length rate variations differ considerably over the solar cycle but even more over a lunar cycle. This is reasonable since the solar perturbation on a synchronous satellite is only 1/4 of the lunar perturbation. Peak-to-peak variations in L and $\dot{L}$ were less than 50,000 feet and 5 feet per second in all cases. The maximum variations during the lunar cycle occur around new moons and to a slightly smaller degree around full moons, and in the Fall of the solar cycle. The length variations shown in Table I result from all satellite position components. We notice that radial, longitudinal, and latitudinal components were in all cases less than 2,500 feet., 0.03°, and 0.005°, respectively, and often considerably less.

To see the effect of omitting daily stationkeeping, the satellite was allowed to move uncorrected for eight days beginning on a day of high and one of low variation. (See Figs. 1 through 4.) An interesting and rather unexpected result is that in both cases, the maxima for the one week runs are the same as for those one day runs with the

larger maxima, whether they occurred at the beginning or end of the week. Thus, one might tolerate stationkeeping once per week rather than daily if the main orbital constraints were peak-to-peak variations in L and $\dot{L}$ of less than 50,000 feet and 5 feet per second.

Another orbital constraint might be a maximum allowable longitudinal drift, say $\Delta\lambda < 0.1°$. The drifts are approximately linear for the first few days, that is, for

$$\text{Oct. 15 to 16,} \qquad \Delta\lambda = 0.026°$$

$$\text{Oct. 15 to 23,} \qquad \Delta\lambda = 0.199°$$

and for

$$\text{Oct. 22 to 23,} \qquad \Delta\lambda = 0.0023°$$

$$\text{Oct. 22 to 30,} \qquad \Delta\lambda = 0.0250°.$$

Thus a constraint of $\Delta\lambda < 0.1°$, might require stationkeeping every

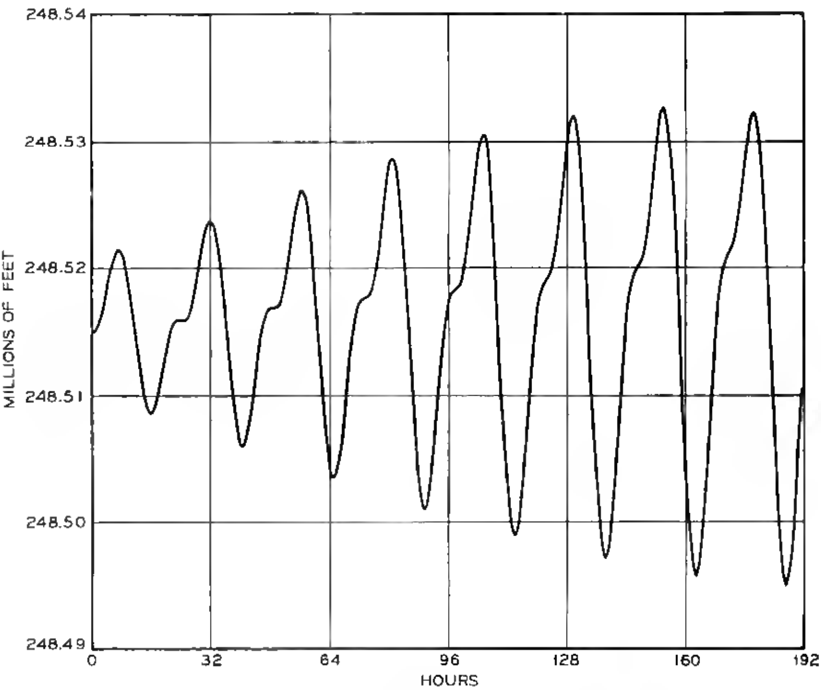

Fig. 3 — Path length variation for eight days starting 10-22:00.

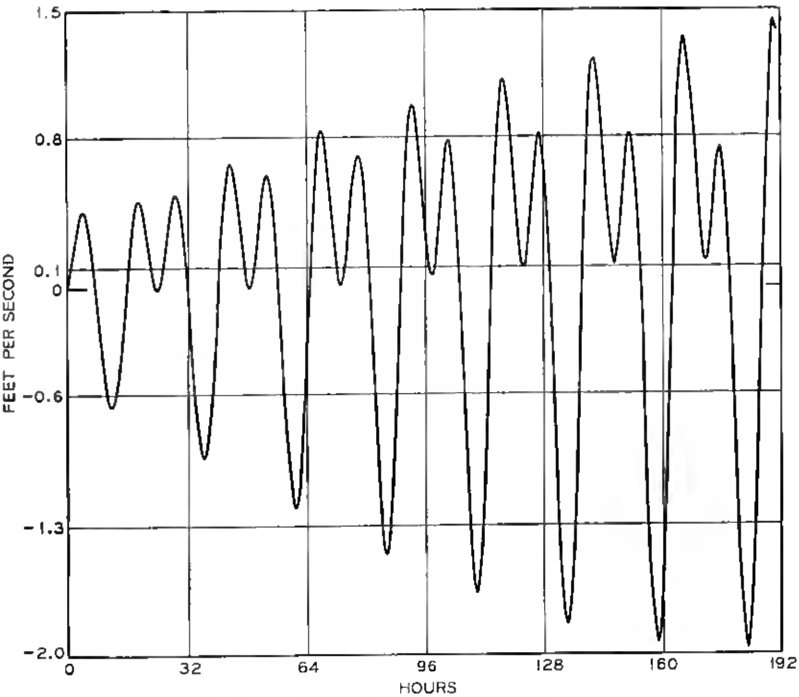

Fig. 4 — Path length rate variation for eight days starting 10-22:00.

3 or 4 days in a period of large variations, although the path length and length rate constaints would not be violated for at least a week.

IV. ACKNOWLEDGMENT

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